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SOCIAL INFLUENCE AND OPINIONS

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In this paper we describe an approach to the relationship between a network of interpersonal influences and the content of individuals' opinions. Our work starts with the specification of social process rather than social equilibrium. Several models of social influence that have appeared in the literature are derived as special cases of the approach. Some implications for theories on social conflict and conformity also are developed in this paper.

KEY WORDS: Conflict, conformity, consensus, influence, network, opinion.

INTRODUCTION

The process of opinion formation rarely boils down to accepting or rejecting the social consensus of others, despite the considerable research (since Asch, 1956) that has been focused on such situations. Typically, individuals form their opinions in a complex social environment in which influential opinions are not only in disagreement, but are also liable to change. Models of how opinions form in such complex circumstances are the topic of our paper.

Models of social influence deal with situations in which individual outcomes are not independent and, therefore, must carefully describe the interactions among the outcomes. Most efforts to describe the interdependence that arises from interpersonal influences have been consistent with the following equilibrium model:

$$\mathbf{Y} = \mathbf{W}\mathbf{Y} + \mathbf{X}\mathbf{B} + \mathbf{U} \tag{1}$$

in which $\mathbf{Y}$ is an $n \times 1$ vector of outcome scores, $\mathbf{W}$ is an $n \times n$ matrix of coefficients giving the effects of each of the n units on the other units, $\mathbf{X}$ is an $n \times k$ matrix of scores on k exogenous variables that may include a constant, $\mathbf{B}$ is a $k \times 1$ vector of coefficients giving the effects of each of the exogenous variables, and $\mathbf{U}$ is an $n \times 1$ vector of residual scores.

Research on this model of social influence includes Duncan, Haller, and Portes (1971), Doreian (1981), Erbring and Young (1979), Burt and Doreian (1982), Marsden and Copp (1986), and Burt (1987). This research has dealt either with methods of estimation or with empirical analyses of special cases of the model. It is our view that insufficient attention has been given to the theoretical foundations and, especially, the deducible implications of the model. Hence, our aim is to take some steps toward deepening and extending the theory that is entailed in this basic model of social influence. The organization of our paper reflects this aim.

First, we ground the model in a broader formal approach—a social network paradigm—that provides elementary simplifying assumptions about the process of opinion change. Previous formal work on the model has begun with the assumption of equilibrium and, thereby, has skirted the issue of the social process. Our paradigmatic approach emphasizes social process and, by doing so, more clearly reveals that the basic model is itself a highly constrained member of a large domain of social influence models. Second, after deriving the basic model from elementary assumptions, we explore its relationship with previous work. Third, and finally, we analyze the implications of the model for the long standing intellectual tensions between investigators who have pursued separate lines of work on social conflict and conformity. Our argument will be that the distinction between conflict and conformity approaches is artificial.

A NETWORK PARADIGM OF OPINION FORMATION

The paradigm can be described broadly in terms of inputs, outputs, and the process linking them. The inputs are exogenous conditions (e.g., individual and group characteristics). The outputs are the settled opinions of a group's members. The paradigm specifies features of the process by which the inputs are transformed into the outputs. The framework of the approach is illustrated in Figure 1 for a group of two persons.

The paradigm divides the process of opinion formation into time periods. In the first time period, the opinions ($\mathbf{Y}$) of n individuals are entirely determined by a set ($\mathbf{X}$) of exogenous variables. That is,

$$\mathbf{Y}_1 = \mathbf{X}_1\mathbf{B}_1, \tag{2}$$

in which $\mathbf{Y}_1$ is an $n \times 1$ vector of opinions, $\mathbf{X}_1$ is an $n \times k$ matrix of scores on k exogenous variables, and $\mathbf{B}_1$ is a $k \times 1$ vector of coefficients giving the effects of each of the exogenous variables.

In the second ($t = 2$) and subsequent ($t = 3, 4, \ldots$) time periods, individuals' opinions continue to be affected by the exogenous variables, but now also are endogenously affected by their own and others' opinions in the immediately preceding period. That is, for $t = 2, 3, 4, \ldots$

$$\mathbf{Y}_t = \alpha_t \mathbf{W}_t \mathbf{Y}_{t-1} + \beta_t \mathbf{X}_t \mathbf{B}_t \tag{3}$$

in which $\mathbf{Y}_t$, $\mathbf{X}_t$, and $\mathbf{B}_t$ have the same definitions as in (2), α_t is a scalar weight of the endogenous conditions, β_t is a scalar weight of the exogenous conditions, and $\mathbf{W}_t$ is an $n \times n$ matrix of the effects of each opinion held at time $t-1$ on the n opinions held at time t. Note that (3) is a recursive definition that applies to all time periods after $t = 1$. Hence, it is not one equation that is described by (3) but rather a set of equations, each member of which applies to a different time period.

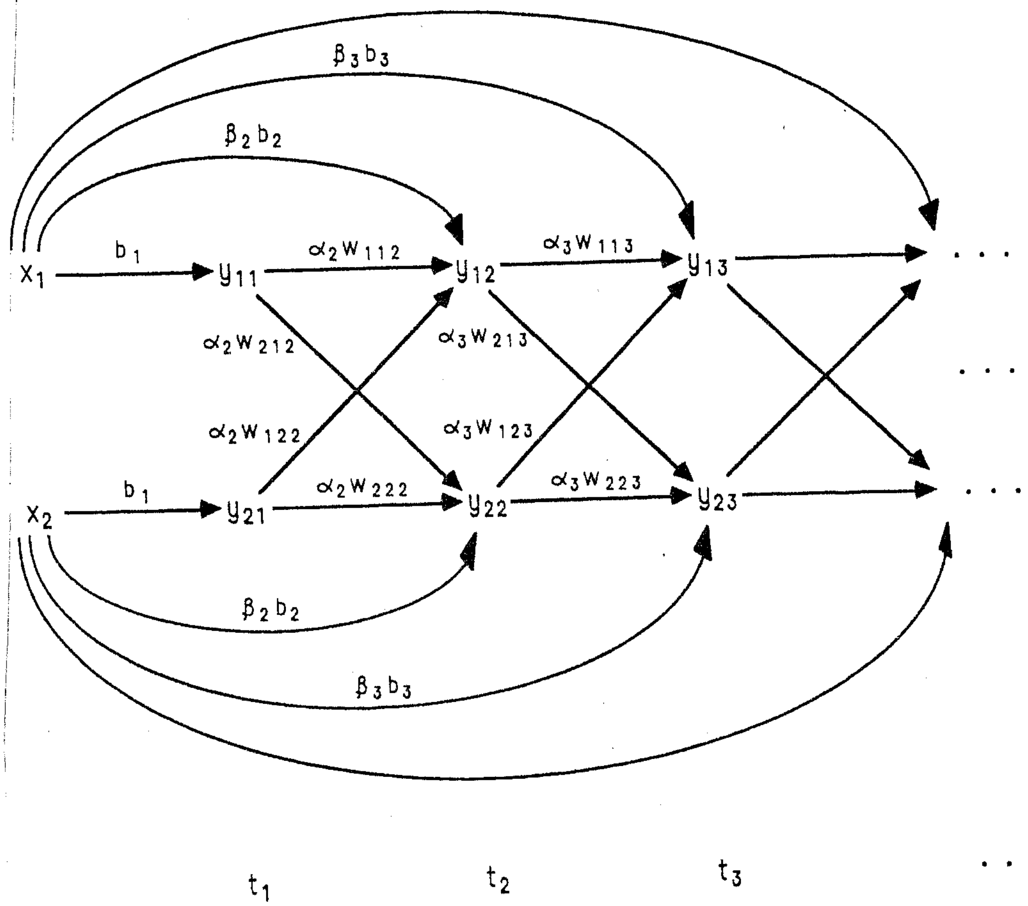

FIGURE 1.

Among this paradigm's assumptions three are fundamental. (1) *Determinism*: There is an assumption that individuals' opinions are completely accounted for by a set of causal variables.[1] Thus, apart from an individual's and other persons' previous opinions, the matrix of exogenous variables, $\mathbf{X}_t$, is viewed as capturing *all* conditions with effects on an individual's opinion.[2] (2) *Decomposability*: There is an analytical assumption that the opinion formation process is decomposable into periods (that need not be of equal length) during each of which linear simultaneous equations supply an accurate prediction of the events that take place.[3] (3) *Continuance*:

[1]Friedkin (1986) distinguishes between power and influence networks depending on whether it is possible or actual effects in **W** that are being dealt with. We are dealing with influence networks in this paper and, thus, have assumed that all of the causal pathways in **W** are active in determining individuals' opinions.
[2]The variables in **X** may be of considerable complexity; for instance, they may be nonadditive and nonlinear combinations of other variables.
[3]A different paradigm m'ight discard the assumption of simultaneity with respect to the influences that are involved in **W**. An alternative paradigm might, for example, rest on an assumption about the prevalence in groups of certain nonrandom sequences of episodes of dyadic or triadic influence.

Finally, there is an assumption that the process of opinion formation continues unless all changes of opinion that might occur have played themselves out.

The elements of this paradigm—α_t, β_t, $\mathbf{B}_t$, $\mathbf{W}_t$, $\mathbf{X}_t$, and $\mathbf{Y}_t$—have been labelled by time period to allow for the possibility that their contents may change over time. Change in these elements may include instabilities in the number (n) of group members or the number (k) of exogenous variables. For a group whose members have attained settled opinions on an issue, the most accurate description of how these opinions were formed might entail changes in *every* time period of *all* of the elements—α_t, β_t, $\mathbf{B}_t$, $\mathbf{W}_t$, and $\mathbf{X}_t$.

Hence, the paradigm permits a highly idiosyncratic opinion formation process: it allows for the possibility that the process unfolds in dramatically different ways in different groups and on different issues. The paradigm also allows for the possibility that the process is one with considerable formal elegance and deducible implications. Indeed, in our view, the important empirical issues that are raised by the paradigm are ones that are related to this tantalizing possibility.

Realizations

The matters that are left open by the paradigm can be resolved either by theory or empirical work. These matters concern the stability and content of α_t, β_t, $\mathbf{X}_t$, $\mathbf{B}_t$, and $\mathbf{W}_t$ during the course of the opinion formation process.

Stability. A model consistent with the paradigm may stipulate that one or more of the conditions are fixed during the course of the process. A model with entirely static conditions would be one in which $\alpha_2 = \alpha_3 = \cdots$, $\beta_2 = \beta_3 = \cdots$, $\mathbf{B}_1 = \mathbf{B}_2 = \cdots$, $\mathbf{W}_2 = \mathbf{W}_3 = \cdots$, and $\mathbf{X}_1 = \mathbf{X}_2 = \cdots$. A more dynamic model would allow for well-behaved change in some of these conditions (see Huckfeldt, Kohfeld, and Likens, 1982; Hannan and Tuma, 1984).

Content. A model also may impose constraints on one or more of the parameters. For instance, one might assert that $\beta_t = 1 - \alpha_t$, where $0 \leq \alpha_t \leq 1$, in order to specify a proportionate impact of exogenous and endogenous variables. With respect to $\mathbf{W}$, one might assume that $0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$ to specify a proportionate impact of influential opinions on individuals' opinions.

MODELS FOR STATIC CONDITIONS

The common feature of the models that we will now derive is that they assume constancy wherever they can. All of the conditions, excepting $\mathbf{Y}_t$, are assumed to be fixed during the process.

The Basic Model

The basic social influence model entails no *a priori* assumptions about content. Dropping the time subscripts on the fixed elements, the process is described simply as follows:

$$\mathbf{Y}_1 = \mathbf{XB} \tag{4}$$

and for $t = 2, 3, \ldots$

$$\mathbf{Y}_t = \alpha \mathbf{W} \mathbf{Y}_{t-1} + \beta \mathbf{XB}. \tag{5}$$

From (4) and (5) we can derive

$$\mathbf{Y}_t = (\mathbf{I} + \alpha\mathbf{W} + \alpha^2\mathbf{W}^2 + \cdots + \alpha^{t-2}\mathbf{W}^{t-2})\beta\mathbf{Y}_1 + \alpha^{t-1}\mathbf{W}^{t-1}\mathbf{Y}_1.$$

When $|\alpha| < 1$ and $|\mathbf{W}^k| < m\mathbf{J}$ for all $k \geq 0$, where $m > 0$ is an arbitrary constant and $\mathbf{J}$ is the $n \times n$ matrix all of whose entries are 1:

$$\lim_{t\to\infty} \alpha^{t-1}\mathbf{W}^{t-1}\mathbf{Y}_1 = 0$$

and when α^{-1} is not an eigenvalue of $\mathbf{W}$

$$\lim_{t\to\infty} (I + \alpha\mathbf{W} + \alpha^2\mathbf{W}^2 + \cdots + \alpha^{t-2}\mathbf{W}^{t-2}) = (\mathbf{I} - \alpha\mathbf{W})^{-1}.$$

Therefore

$$\lim_{t\to\infty} \mathbf{Y}_t = (\mathbf{I} - \alpha\mathbf{W})^{-1}\beta\mathbf{Y}_1$$

demonstrating that

$$\mathbf{Y}_\infty = \lim_{t\to\infty} \mathbf{Y}_t$$

exists and that $\mathbf{Y}_\infty$ satisfies the reduced-form equations

$$\mathbf{Y}_\infty = [\mathbf{I} - \alpha\mathbf{W}]^{-1}\beta\mathbf{Y}_1 \tag{6}$$

$$\mathbf{Y}_\infty = [\mathbf{I} - \alpha\mathbf{W}]^{-1}\beta\mathbf{XB}. \tag{7}$$

The first of these reduced-form equations expresses the settled opinions of a group, $\mathbf{Y}_\infty$, in terms of initial opinions, $\mathbf{Y}_1$; the second expresses the settled opinions in terms of **XB**. Multiplying (7) through by $[\mathbf{I} - \alpha\mathbf{W}]$, we get

$$[\mathbf{I} - \alpha\mathbf{W}]\mathbf{Y}_\infty = \beta\mathbf{XB}$$

or

$$\mathbf{Y}_\infty = \alpha\mathbf{W}\mathbf{Y}_\infty + \beta\mathbf{XB}.$$

For a disturbance term that consists exclusively of contributions of unanalyzed causal effects, we simply partition the **X** and **B** matrices. That is, let $\mathbf{X}_*$ contain n observed scores on a subset of the variables in **X**, let $\mathbf{B}_*$ contain the coefficients associated with these variables, and let $\mathbf{U} = \beta\mathbf{XB} - \beta\mathbf{X}_*\mathbf{B}_*$.[4] Hence,

$$\mathbf{Y}_\infty = \alpha\mathbf{W}\mathbf{Y}_\infty + \beta\mathbf{X}_*\mathbf{B}_* + \mathbf{U}. \tag{8}$$

We may simplify further by subsuming either or both of the weights, α and β, into their respective coefficient matrices, thus formally yielding (1).

Duncan, Haller, and Portes (1971), Erbring and Young (1979), Doreian (1981), Burt and Doreian (1982), Marsden and Copp (1986), and Burt (1987) have dealt with special cases of (7). For example, from (7) we get a model of dyadic influence that appeared in Duncan et al. (1971): When $n = 2$

$$\mathbf{Y}_\infty = [\mathbf{I} - \alpha\mathbf{W}]^{-1}\beta\mathbf{XB} = \alpha\mathbf{W}\mathbf{Y}_\infty + \beta\mathbf{XB}$$

[4]It should be carefully noted that this partition does not necessarily produce a disturbance term that is independent of the observed variables in **X**.

entails the two equations:

$$y_{1\infty} = (\alpha w_{11})y_{1\infty} + (\alpha w_{12})y_{2\infty} + \beta \sum_k b_k x_{1k}$$

$$y_{2\infty} = (\alpha w_{21})y_{1\infty} + (\alpha w_{22})y_{2\infty} + \beta \sum_k b_k x_{2k}$$

Along with the subsumption of α and β into their respective coefficient matrices, Duncan et al. set the diagonal of $\mathbf{W}$ to zero, so that the terms involving w_{11} and w_{22} drop out of the equations.[5] $\mathbf{W}$'s with diagonals set to zero are common in work on the basic model. The other common constraint on $\mathbf{W}$ is a normalization such that $0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$.

OTHER SPECIAL CASES

Other special cases of the basic model include the linear discrepancy model (Hunter, Danes, and Cohen, 1984), the peer effects model (commonly appearing in stratification research), and the consensus models of French (1956), Harary (1959) and DeGroot (1974).[6]

Linear Discrepancy Model

From (6) we get a model that has appeared frequently in experimental research, most recently in the work of Hunter, Danes, and Cohen (1984). Consider again the two equations of an $n = 2$ system:

$$y_{1\infty} = (\alpha w_{11})y_{i\infty} + (\alpha w_{12})y_{2\infty} + \beta \sum_k b_k x_{1k}$$

$$y_{2\infty} = (\alpha w_{21})y_{1\infty} + (\alpha w_{22})y_{2\infty} + \beta \sum_k b_k x_{2k}$$

From (4), $\sum_k b_k x_{1k} = y_{11}$. If $0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$ (there is a proportionate contribution of influential opinions) and if

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} \\ 0 & 1 \end{bmatrix}$$

(1 has no influence on 2), then $\beta = 1 - \alpha$ (there is a proportionate contribution of exogenous and endogenous conditions), $y_{21} = y_{2\infty}$ (2's initial opinion is fixed), and

[5]Duncan et al. permit $\mathbf{B}$ to differ for each individual. Our model does not deal with this situation. Because $\mathbf{X}$ may involve multiplicative conditions, our position is not highly restrictive and is, in fact, supported by Duncan et al.'s findings of small individual differences in the effects of the exogenous variables.

[6]Formal theories of opinion formation that cannot be subsumed and do not have an especially close formal resemblance to the present model include Abelson (1964), Doreian (1979), Johnsen (1986), and Marsden (1981). For a normative model of opinion formation that is consistent with the present one see Wagner (1978). For stochastic models concerned with the likelihood that persons will agree across a large set of issues see Kelly (1981) and Friedkin (1986).

the prediction for 1 is

$$y_{1\infty} = \alpha w_{11} y_{1\infty} + \alpha w_{12} y_{21} + \beta y_{11}$$

or

$$(y_{1\infty} - y_{11}) = [\alpha w_{12}/(1 - \alpha w_{11})](y_{21} - y_{11})$$

for $y_{21} = y_{2\infty}$. In this case, in short, the change in member 1's opinion is a constant proportion of the discrepancy between 1 and 2's initial opinions.

Peer Effects Model

When the entries in row i of $\mathbf{W}$ consist of m_i entries $1/m_i$ and $n - m_i$ entries 0:

$$\alpha \mathbf{W} \mathbf{Y}_\infty = \alpha \mathbf{M}$$

where $\mathbf{M}$ is an $n \times 1$ vector containing, for each individual, the mean opinion of those persons whose opinions have had a *direct* effect on the individual's opinion [n.b., such influentials need not be in face-to-face contact with the focal individual]. Model [8] now simplifies to the peer effects model

$$\mathbf{Y}_\infty = \alpha \mathbf{M} + \beta \mathbf{X}_* \mathbf{B}_* + \mathbf{U}.$$

It is this model that has most frequently been employed in efforts to estimate the impact of interpersonal influences on individual's opinions. However, use of the model has been criticized because of the absence of any theoretical foundations (for example, see the comments of Spenner and Featherman, 1978). The theoretical foundations that we have provided indicate that the model may be used when an investigator can assume an equality of the direct interpersonal effects on an individual.[7]

The present foundations for the model indicate that a noteworthy effect of $\mathbf{M}$ (i.e., the mean opinion of those persons whose opinions have directly affected an individual) does not necessarily indicate the impact of a group norm. The mean of influential opinions enters into the model strictly as a resultant, or constructed, variable that may have a weight in the *prediction* of individuals' opinions. For explanatory purposes, what is relevant is that the weight estimated for $\mathbf{M}$ is equivalent to the causal weight of the *set* of endogenous influences that have had a hand in determining individuals' opinions. It is seriously misleading to interpret a noteworthy effect of the mean of "peer" opinions as indicative of any conformance to a norm, because such an effect also is consistent with normatively ambiguous situations in which settled opinions are scattered.

Group Consensus Models

The basic model subsumes as limiting cases the work of French (1956), Harary (1959), and DeGroot (1974) who were concerned with the structural conditions

[7]The model also is reasonable if an investigator can divide the persons who have influenced the subject into subsets in each of which there is an equality of direct effects.

under which group consensus might emerge. These special cases assume that $\mathbf{W}$ is normalized ($0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$) and that $\beta = \delta(1-\alpha)$. Given the limit

$$\lim_{\alpha \to 1^-} (1-\alpha)[\mathbf{I} - \alpha\mathbf{W}]^{-1} = \mathbf{W}^{\infty},$$

see the Appendix for a proof, (6) and (7) may be transformed as follows:

$$\begin{aligned} \mathbf{Y}_{\infty} &= [\mathbf{I} - \alpha\mathbf{W}]^{-1}\beta\mathbf{Y}_1 \\ &= (1-\alpha)[\mathbf{I} - \alpha\mathbf{W}]^{-1}\delta\mathbf{Y}_1 \underset{\alpha \to 1^-}{\longrightarrow} \delta\mathbf{W}^{\infty}\mathbf{Y}_1 \qquad (6a) \\ &= (1-\alpha)[\mathbf{I} - \alpha\mathbf{W}]^{-1}\delta\mathbf{XB} \underset{\alpha \to 1^-}{\longrightarrow} \delta\mathbf{W}^{\infty}\mathbf{XB} \qquad (7a) \end{aligned}$$

French and Harary's models stem from (6a). French stipulates that all nonzero effects of prior opinions on an individual's opinion are of equal magnitude. Harary slightly relaxes French's assumption by allowing an individual's own prior opinion to have a weight that is different from the weight of other persons' opinions. DeGroot's model imposes no additional constraints on $\mathbf{W}$; thus, his model is directly represented by (6a).[8]

IMPLICATIONS FOR SOCIAL CONFORMITY AND CONFLICT

Because networks of social influence are involved in many phenomena of concern to social scientists, network models of social influence have a broad array of implications. However, we are not aware of any programs of empirical work that have sought to evaluate the deduced implications of the basic model. To illustrate the possibilities of the model as a hypothesis generator, we present some of its implications for the study of social conformity and conflict.

Background

Prevalently held opinions (these may be preferences, expectations, views, beliefs, convictions, persuasions, or sentiments) occupy a prominent place in sociological theory. For our present task, it is not important whether these opinions are the constituent elements of mores, roles, ideologies, policies, or standard operating procedures. What is important is the idea of a normative order consisting of opinions that have modest variances, if they are not consensually held by the members of a group. Recurrent behaviors can be explained on the basis of a normative order when the order is durable and individuals' identifications with it are strong.

Movement of variant opinions toward normative opinions is a central characteristic of theories of social conformity. The evidence is abundant that most individuals find it difficult to maintain a highly idiosyncratic opinion on an issue when the other members of their groups have relatively uniform opinions on the matter. Hence, it has been shown that, in the context of a normative order, a frequent outcome of social influence processes is the maintenance and reinforcement of the order.

[8]One might add a constraint that $w_{ii} > 0$. This constraint would stipulate that individuals' prior opinions must have some weight (even if negligible in some cases) on their subsequent opinions. Analytically, $w_{ii} > 0$ would assure that as $t \to \infty$ $\mathbf{W}^t$ converges to $\mathbf{W}^{\infty}$.

In reaction to the heavy emphasis placed on normative orders in sociological theory, it has been repeatedly and correctly pointed out that these orders do not account for persistent disagreements and social conflicts. Theories that rely on conditions of normative order and processes of social conformity have been criticized as presenting an "oversocialized" view of individuals (Wrong, 1961). Also consistent with such criticism are arguments that the logical coherence of normative orders has been overstated and that, consequently, such orders do not provide a satisfactory explanation of recurrent behaviors (Merton, 1976).

From a social conflict perspective, interpersonal influences may or may not contribute to a reduction of individual differences and, only in special circumstances, do they result in a consensus of opinion. Conflict-oriented theories pursue explanations of (a) the variance in individuals' opinions, (b) enduring patterns of disagreement, (c) the formation of agreements among individuals with heterogeneous interests and enduring social differences, and (d) the negotiation of action and meaning in normatively ambiguous situations.

Social Influence Mechanisms

These two perspectives, which have been warring for decades over the proper direction of sociological imagination, do not require different theories about the mechanism of social influence. The basic model of social influence is consistent with both perspectives, as we now will demonstrate. For the analysis we assume that $\mathbf{W}$ is normalized ($0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$) and that $\beta = \delta(1-\alpha)$ where $0 < \alpha < 1$ and $0 \leq \delta$.

From (6) we get

$$\mathbf{Y}_\infty = \delta \mathbf{V} \mathbf{Y}_1$$

where

$$\mathbf{V} = (1-\alpha)(\mathbf{I} - \alpha \mathbf{W})^{-1}.$$

An entry, v_{ij}, in $\mathbf{V}$ gives the *total* (i.e., direct and indirect) relative effect of group member j on the final opinion of group member i. The magnitudes of these coefficients will range from 0 to 1. The values in the ith row of $\mathbf{V}$ will sum to 1 and represent the manner in which the interpersonal influence on i's final opinion is distributed among group members.

Now consider one person (i) and the equation for that person's final opinion ($y_{i\infty}$, an entry in $\mathbf{Y}_\infty$) in terms of the entries in $\mathbf{V}[v_{ij}]$ and $\mathbf{Y}_1 = [y_i, y_2, \ldots y_n]^{\mathrm{T}}$

$$y_{i\infty} = \delta \sum_j v_{ij} y_j \tag{9}$$

i's *final* opinion is predicted to be a weighted average of the *initial* opinions of the n members of i's group. A more elaborate and theoretically suggestive form of (9) is

$$y_{i\infty} = \underbrace{\delta[(1-v_{ii})m + v_{ii}y_i]}_{[a]} + \underbrace{\delta\left[\sum_j v_{ij}(y_j - m)\right]}_{[b]} \qquad i \neq j \tag{10}$$

in which m is the mean initial opinion of all members in the group *other than* member i (i.e., $(\sum_j y_j - y_i)/(n-1)$). Part [a] of (10) consists of $(1-v_{ii})m + v_{ii}y_i$ which is a weighted average of i's initial opinion and the mean of other members' initial opinions. If the effect of this part of (10) dominates, then *any discrepancy between i's and the group's opinions will be reduced by a shift of i's initial opinion toward that of the group's initial opinion so long as i accords some weight to the opinion of at least one other member of the group. The larger the total interpersonal effects of other members on i, the closer will i's opinion be to the mean of other members' initial opinions.* This is the fundamental prediction of Festinger's (1954) Theory of Social Comparison Processes. It also is the hypothesis underlying much of the social conformity research (see Moscovici, 1985) that has followed the experiments of Asch (1956).

The social conformity implications of the model are most obvious when all the members of i's group *except for member i* hold an identical initial opinion. In this circumstance (10) simplifies to

$$y_{i\infty} = \delta[m(1-v_{ii}) + v_{ii}y_i]$$

and the social conformity prediction, stated above, holds without qualification.

The other part of (10) consists of $\sum_{j\neq i} v_{ij}(y_j - m)$ which is determined by the relative influence of each of the other members on i. This term captures the group members' differences of opinion and the bearing of these differences on the final opinion of i. If the effect of this part of (10) dominates, then *an individual is confronted by a normatively ambiguous situation of heterogeneous influential opinions. The outcome for the individual is determined by the differences in the total (direct plus indirect) effects of the separate members on the individual.* In a situation of social conflict, the prediction of individuals' settled opinions will require data on the *pattern* of direct interpersonal influences in a group and an analysis of the consequences of this pattern in terms of the total (direct and indirect) interpersonal influences of the members on each other.

Our argument is that there is one process of social influence that is implicated in theories of normative order and social conflict. Our analysis suggests that the applicability of a theory of normative order or of social conflict to a particular situation depends on the variation of group members' initial opinions on issues. On the one hand, the mean of other group members' initial opinions will accurately predict an individual's final opinion on an issue if the variance of their initial opinions is low *and* the effects of interpersonal influences are large. On the other hand, there will be no strong tendency for individuals to settle on opinions that are close to the mean of their group's initial opinions when the initial variances are large and there is substantial differentiation in interpersonal influences *or* when the effects of interpersonal influences are weak.

Substantial variances of initial opinions arise from social heterogeneity on exogenous conditions. Social conflict theories accordingly emphasize either an analysis of this antecedent heterogeneity or the intervening structure of interpersonal influences that happen to transmit, rather than to importantly mitigate, disagreements.

SUMMARY

We have presented an account of the process of opinion formation in which the effects of a particular set of conditions—the opinions of other persons—have been displayed in more detail than any other determinants. We have characterized this account as a paradigm because it rests on a primitive set of assumptions about how individuals' opinions are formed and how, in particular, the opinions of other persons enter into the process of opinion formation.

After describing this paradigm, we derived a static conditions model that has appeared frequently in formal approaches to social influence. We have two reasons for separating the discussions of paradigm and model. First, this separation makes it plain that the model is a member of a large class of models that are consistent with a broad formal approach. Second, this separation serves to highlight the primitive assumptions of the paradigm; these primitive assumptions merit scrutiny because their revision may involve the development of *qualitatively* different ways of describing the process of opinion formation.

We have sought to somewhat deepen and extend the formal theory that is implicated in the basic model of social influence. Several models that have appeared in the literature were derived as special cases. From the perspective of the previous empirical work on social influence, the most important of these special cases is the common peer effects model in which the mean opinion of alters is employed to predict an individual's opinion. The present theoretical foundations for the peer effects model generate three caveats: (a) "Peers" must include those persons whose opinions have a *direct* effect on the individual's opinion regardless of whether such persons are in face-to-face communication with the individual. (b) The model is accurate only when the direct effects of the peers are equal. (c) An effect of the mean opinion of peers indicates the weight of the social influence process in determining individuals' opinions and ought not be reified as the effect of any social norm.

Finally, we have shown that the static conditions model is applicable to both circumstances of marked social conflict and consensus. We have suggested that social conflict and social conformity behaviors simultaneously exist in any group. Since their relative importance is likely to vary across groups and issues, arguments as to which is the more important in a particular group only serve to distract attention from the more fundamental job of constructing better theory.

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APPENDIX

Here we show the following result, which is invoked in our discussion of group consensus models:

THEOREM. *Let W be an $n \times n$ stochastic matrix for which*

$$\lim_{t\to\infty} W^t = W^\infty \tag{A1}$$

exists. Then

$$\lim_{\alpha\to 1^-} (1-\alpha)(I-\alpha W)^{-1} = W^\infty. \tag{A2}$$

PROOF. Since the entries of every matrix W^k, $k \geq 0$, lie between 0 and 1 and we may take $0 < \alpha < 1$, the infinite series expansion

$$(1-\alpha)(I-\alpha W)^{-1} = (1-\alpha)\{I + \alpha W + \alpha^2 W^2 + \alpha^3 W^3 + \cdots + \alpha^k W^k + \cdots\} \tag{A3}$$

converges absolutely. We also have the absolutely convergent series

$$\begin{aligned} W^\infty &= (1-\alpha)(1-\alpha)^{-1} W^\infty \\ &= (1-\alpha)\{W^\infty + \alpha W^\infty + \alpha^2 W^\infty + \cdots + \alpha^k W^\infty + \cdots\}. \end{aligned} \tag{A4}$$

Subtracting corresponding sides of (A4) from (A3) and taking absolute values we obtain the inequality

$$\begin{aligned} |(1-\alpha)(I-\alpha W)^{-1} - W^\infty| \leq (1-\alpha)\{&|I - W^\infty| + \alpha|W - W^\infty| \\ &+ \alpha^2|W^2 - W^\infty| + \cdots + \alpha^k|W^k - W^\infty| + \cdots\}. \end{aligned} \tag{A5}$$

Given any $\epsilon > 0$ we want to obtain an $\alpha(\epsilon)$ such that for all α satisfying $\alpha(\epsilon) < \alpha < 1$ the left side of (A5) is less than ϵJ, where J is the $n \times n$ matrix all of whose entries are 1. If this can always be done the theorem will be proved.

Now, because of (A1), for any $\delta > 0$ there is an integer $N(\delta) > 0$ such that $|W^k - W^\infty| < \delta J$ for all $k \geq N(\delta)$. Denoting the left side of (A5) by LS, we have by this that

$$\begin{aligned} \text{LS} < (1-\alpha)\{&|I - W^\infty| + \alpha|W - W^\infty| + \cdots + \alpha^{N(\delta)-1}|W^{N(\delta)-1} - W^\infty| \\ &+ \alpha^{N(\delta)}(1 + \alpha + \alpha^2 + \cdots)\delta J\}. \end{aligned} \tag{A6}$$

Let $\beta > 0$ be such that $|W^k - W^\infty| < \beta J$ for all $0 \leq k \leq N(\delta) - 1$. Then (A6) becomes

$$\text{LS} < (1-\alpha)(1-\alpha^{N(\delta)})(1-\alpha)^{-1}\beta J + (1-\alpha)\alpha^{N(\delta)}(1-\alpha)^{-1}\delta J$$

or

$$\text{LS} < [(1-\alpha^{N(\delta)})\beta + \alpha^{N(\delta)}\delta]J. \tag{A7}$$

By choosing $\delta < \epsilon/2$ we obtain $\alpha^{N(\delta)}\delta < \epsilon/2$. By choosing α sufficiently close to 1 we can also obtain $1 - \epsilon/(2\beta) < \alpha^{N(\delta)}$, as follows. If $\epsilon/(2\beta) \geq 1$ there is no constraint on α. If $\epsilon/(2\beta) < 1$ we take $\alpha > [1 - \epsilon/(2\beta)]^{1/N(\delta)} \equiv \alpha(\epsilon)$. Thus, we can obtain $(1 - \alpha^{N(\delta)}) < \epsilon/(2\beta)$ for $\alpha(\epsilon) < \alpha < 1$, and for such α we obtain from (A7)

$$\text{LS} < [\epsilon\beta/(2\beta) + \epsilon/2]J = \epsilon J,$$

or

$$|(1-\alpha)(I - \alpha W)^{-1} - W^{\infty}| < \epsilon J, \tag{A8}$$

which proves (A2).

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