
8. THE STRUCTURE OF SOCIAL SPACE

Abstract: The distribution of actors in social space governs the probability of interpersonal attachments which, in turn, governs the distribution of interpersonal influence. To describe the structure of social space, I develop the concept of ridge structure, which is an object in social space comprised of sequentially overlapping and densely occupied regions of social space. Because the probability of an interpersonal attachment increases with the proximity of actors in social space, a ridge structure is associated with sequentially intersecting cohesive subsets of actors. In this chapter, I support the concept of ridge structure with both theoretical and empirical results.

The macro-structure of attachments allows flows of interpersonal influence to penetrate into more or less distant regions of a social space and to accumulate so that certain actors come to have a substantial effect on system outcomes. I will argue that bridges typically do not provide the most important foundations of such reachability and impact. To describe the more typical foundations of reachability and interpersonal influence in complexly differentiated social spaces, I develop the concept of ridge structure. A ridge structure is an object in social space (i.e., a type of social manifold) consisting of sequentially overlapping and densely occupied regions of social space. Because the probability of an interpersonal attachment increases with the proximity of actors in social space, ridge structures produce sequentially intersecting cohesive subsets of actors. In a ridge structure, each actor is embedded in a cohesive environment of interpersonal attachments which has, in turn, two consequences. First, the mean indegree of actors is high so that most self-weights are low and influence processes can lead to agreements. Second, structural connectivity is established on a robust basis so that the cessation of particular attachments, and the death and mobility of individuals, do not substantially alter the net effects of social positions.

First, I will point to previous literature (especially balance theory) in which the idea of a ridge structure has appeared; I show that transitivity implies a ridge structure among cliques and a macro-structure of attachments in which each clique corresponds to a social position occupied by structurally equivalent actors. Second, in the faculties of science at Chicago and Columbia, I show that ridge structures exist that connect actors who are located in distant social positions. I show that, although bridging ties can be important in linking actors who are located in different ridge structures, the more robust basis of macro-level connectivity is a ridge structure that penetrates into various areas of social space and that includes a relatively large fraction of the population of actors.
8.1 Segregated Structures, Ridge Structures, and Bridges

It is an axiom of structural analysis that the probability of a social tie is negatively associated with the distance between the positions of actors in social space (Blau 1994; Laumann and Knoke 1986). I have documented such associations in the previous chapter. The association between interpersonal attachment and social distance has certain implications for the overall pattern of attachments in a population and, in turn, the structure of interpersonal influences.

8.1.1 Bridges in Segregated Structures

If the distance between each social position is sufficiently large, then nearly all of the ties will occur among those actors who occupy the same positions, and only a few ties will span the boundaries of these subgroup formations. When inter-positional ties are rare, each such tie is likely to be a local bridge (i.e., a tie between two actors who are not joined by any semipaths of length two). More generally, where social positions are segregated into distant regions of the social space, and where the social positions within each of these regions are in close proximity, the expected macro-structure is a set of subgroup formations or, more precisely, position-clusters that are connected (if at all) by a small number of bridges:

Figure 8.1. Segregated Macro-Structure

Granovetter (1973) argues that the bridges between such subgroups (position-clusters) are likely to be weaker ties than those within the subgroup formations. Suppose that the strength of an interpersonal attachment is equated with the proximity of the attached actors in social space, and, therefore, with the probability of the attachment under the conditions which have defined the social space. With this definition, an
anomalous situation is generated by a strong local bridge if either actor who is involved in the bridge has at least one other strong attachment. Assume that actor $i$ is strongly tied to actor $j$ on a local bridge and that $j$ also is strongly tied to some other actor $k$. This implies that the social distances between $i$, $j$ and $k$ are small, and that $i$ is likely to be tied to $k$. But if the likely tie from $i$ to $k$ occurs, then the tie from $i$ to $j$ is not a local bridge. Thus, ties which are not strong (i.e., ties between actors who occupy distant positions in social space), because they do not entail such an anomalous closure, are more likely to be bridges.\(^{35}\)

### 8.1.2 Ridge Structures

Drawing on balance theory, and specifically the tendency to transitive closure, Granovetter (1973) argues that segregated macro-structures are to be expected and that, therefore, bridging ties are theoretically important phenomena — the key structural foundations of connectivity in differentiated populations. I now develop an argument that a segregated macro structure is a special case of a broader class of macro-structures — ridge structures — and that in many of the macro-structures that belong to this broader class, bridges do not provide the key structural foundations of macro-level connectivity.

To introduce the idea of a ridge structure, consider a contour map of a mountain range on which lines are drawn that connect points on the land surface that are at a certain elevation. Setting the elevation of the contour lines sufficiently high will serve to reveal the mountain peaks, but will provide a poor visualization of the overall structure of the mountain range. If we set the elevation of the contour lines somewhat lower, we may reveal a structure of elevated ridges along which we can traverse (trace a contour line) and move between distant points in the mountain range. These ridge structures may entail more or less steep negative gradients, and, where these gradients are sufficiently steep, we can conceptualize the areas between the ridges as gaps or holes. Any phenomenon that spans such gaps (e.g., a bridge, an agreement) is a boundary spanning phenomenon. A local bridge may connect different ridge structures or connect parts of the same ridge structure, but it does not constitute the structure.

Now if the elevation of a pair of actors is defined in terms of their proximity in social space, then a social structure may be described in terms of the high elevation (highly proximate) subgroup formations that occur in a population. By the same token, a high elevation ridge structure also may occur in which actors in different parts of the social structure are connected by one or more chains of sequentially overlapping, densely occupied, regions of social space. Such a high-elevation ridge structure is based on the same structural foundations as the "peaks" that occur in this structure — high levels of proximity in social space and probabilities of interpersonal attachment.

Social differentiation simultaneously establishes both "peaks" and "ridges", and it is the configuration of the "ridge" structure that I take to be a key (frequently occurring) foundation for the development of shared agreements between actors who located in different parts of complexly differentiated social structures. A ridge structure is a type of social manifold (i.e., an object in social space) consisting of sequentially overlapping, densely occupied, regions of social space. Because the probability of an interpersonal attachment increases with the proximity of actors in social space, ridge structures are
associated with a framework of sequentially contiguous zones (overlapping neighborhoods or intersecting social circles) of structural cohesion. The framework of interpersonal attachments governs the distribution of interpersonal influence, and it is the structure of this framework that determines the relative influence of social positions. In this view, social differentiation does not simply engender cohesive subgroup formations; the concept of discrete, cohesive, subgroups is an oversimplification that I discard and replace with the more general concept of ridge structures.36

8.2 Blockmodels, Link-Pin Organizations and Social Circles

The concept of ridge structure has appeared previously in the literature in different guises. I now pull this related work together and show how different theoretical analyses have entertained the idea of a ridge structure. This related work includes White, Breiger and Boorman's concept of a blockmodel, Likert's analysis of "linking pins", Alba and Kadushin's concept of "social circles", and generalizations of Harary and Cartwright's balance theory.

8.2.1 Ridge Structures and Blockmodels

I introduced the concept of a blockmodel in Chapter 1. A blockmodel is an image of the macro-structure of a social network in which the points are social positions (or position-clusters) and the lines are defined on the basis of the density (or average value) of the network ties among either the occupants of the same social position or two different social positions. Let \( M = m_{ij} \) be a valued network, where \( m_{ij} \) is the value of the line \( (i \rightarrow j) \) from actor \( i \) to actor \( j \). When the actors are partitioned into position-clusters, the macro-structure of the social relation \( M \) can be described by a \( k \times k \) blockmodel, \( M^* = m^*_{ij} \), where \( k \) is the number of distinct social positions (position-clusters), and \( m^*_{ij} \) is the mean value of the social relation \( M \) among actors in the same position-cluster (for \( i = j \)) or between actors in two different position-clusters (for \( i \neq j \)). From \( M^* \), a simplified image of the blockmodel can be developed, that is, \( \hat{M}^* = \hat{m}^*_{ij} \), in which the adjacency of social positions is defined in terms of a threshold value of \( m^*_{ij} \):

\[
\hat{m}^*_{ij} = \begin{cases} 
1 & \text{if } m^*_{ij} > e \\
0 & \text{if } m^*_{ij} \leq e 
\end{cases}
\]  

(8.1)

For example, if \( M \) is an adjacency matrix (i.e., \( m_{ij} \) has the value of zero or one), then blockmodel image \( \hat{M}^* \) indicates which of the network densities within and between position-clusters are above the threshold value \( e \): if the threshold is \( e > 0 \), then a line will exist in the image given at least one line in \( M \) from a member of position \( i \) to a member of position \( j \) (the so-called zeroblock criterion); and if the threshold is sufficiently close to 1, then a line will exist in the image only if there is a line in \( M \) from each of the members of position \( i \) to all of the members of position \( j \) (the so-called oneblock criterion). Typically, when \( M^* \) is a density matrix, a framework of linkages or bonds between social positions is obtained by setting the threshold density to an intermediate value
between zero and one. However, if the actors in the position-cluster are structurally equivalent in \( \mathbf{M} \), then the density of ties within and between the position-clusters must be either zero or one.

Consider a population of actors that is partitioned into social positions in which actors \( i \) and \( j \) (\( i \neq j \)) jointly occupy a social position only if they are structurally equivalent, where structural equivalence requires identical profiles of sent and received ties on a social relation, \( \mathbf{M} = \mathbf{m}_{ij} \), i.e.,

\[
m_{ii} = m_{jj}, \quad m_{ij} = m_{ji}, \quad m_{ik} = m_{jk}, \quad \text{and} \quad m_{ij} = m_{ij}
\]

for all \( k \neq i, j \). In such a population, a blockmodel image of the macro-structure of the social relation can be constructed on a zeroblock or oneblock criterion with the same result, because the densities of ties within and between social positions will be either zero or one. The existence of a tie between two actors \( i \) and \( j \) also implies the existence of a tie to actor \( j \) from all the actors who are structurally equivalent to \( i \); and since actor \( j \) has received a tie, all actors who are structurally equivalent to \( j \) must also receive a tie from \( i \); and so forth. Hence, to the extent that the actors within a set of empirically defined position-clusters are structurally equivalent on a social relation, the densities within and between such position-clusters should be close either to one or zero.\(^{37}\)

In short, a ridge structure of social ties is entailed in a macro-structure that consists of social positions in which the joint occupants of each social position are structurally equivalent on the social relation.\(^{38}\) All of the social ties might be within social positions, or there might be a framework of dense social positions and interfaces between certain positions. A social position within which the density of ties is zero is not precluded. However, zero-density positions are exceedingly unlikely; if the probability of a social tie increases as the social distance between two actors declines, then a jointly occupied social position is likely to be dense.

If there are no jointly occupied positions in social space, then a position-cluster that is defined in such a space will consist of more or less structurally equivalent actors. In this case, it cannot be assumed that a valued line from an actor in position-cluster \( A \) to an actor in position-cluster \( B \) implies lines with an identical value from all actors in \( A \) to all actors in \( B \). However, to the extent that actors are proximal in social space, the pattern of their ties will be similar.

\subsection*{8.2.2 "Linking Pins" in Formal Hierarchies}

The concept of a ridge structure also appears on the classic work of Likert (1961), as part of an analysis of authority hierarchies in organizations. This work is especially noteworthy because Likert argued, in effect, that ridge structures are crucial to the coordination of organizational activities (which also is my argument).

Consider a formal hierarchy of authority (a tree) in which each member has only one immediate supervisor and in which ties of interpersonal communication and influence are restricted to the lines of formal authority. Such a network is unipathic, i.e., there is only one path (or semipath) connecting each pair of actors. The pairs of actors who are in direct contact have no shared contacts. The actors who are not in direct contact have either one shared contact (an intervening supervisor) or none at all. Each tie is bridge and
every supervisor is a *liaison* (cut point). Severing a tie between any two actors or removing a supervisor disconnects the network.

**Linking pins.** Likert (1961) argued that in effective organizations this hierarchical arrangement is modified in two ways. First, interpersonal influences are symmetric rather than anti-symmetric. Although the influences between a supervisor and subordinate may be markedly unbalanced in favor of the supervisor, in effective organizations subordinates usually have some influence on their immediate supervisors. Because of the hierarchical arrangement of authority, all supervisors (except for the supreme authority) are also subordinates. Likert argued that the effectiveness of supervisors depends on their ability to influence their immediate supervisors:

> Subordinates expect their supervisors to be able to exercise an influence upward in dealing with problems on the job and in handling problems which affect them and their well-being. (p. 113)

Hence, effective organizations tend to be ones in which influences between subordinates and supervisors are mutual. Second, effective organizations entail additional lines of influential communication among the immediate subordinates of each supervisor. Likert states:

> Effective groups with high group loyalty are characterized by efficient and full communication and by the fact that their members respect each other, welcome attempts by the other members to influence them, and are influenced in their thinking and behavior when they believe that the evidence submitted by the other members warrants it. (p. 114)

Likert (p. 113) illustrates these structural arrangements as in Figure 8.2. Each triangle represents a work group of size greater than two, consisting of a supervisor and his or her immediate subordinates. The supervisor of each such work group operates as a linking pin because, being a member of two work groups, he or she serves to connect the groups. Thus, the "link-pin" organization eliminates all bridges. However, the liaison (cut point) position of the supervisors is maintained.
Short-circuits. Lateral extensions of influential communication raise the problem of a subordinate encountering two equivalent superiors. Vertical extensions of influential communication do not raise this problem in as severe form. I refer to these additional vertical communications as short circuits. Likert (1961, p. 115) introduces the idea of short circuits as follows:

To help maintain an effective organization, it is desirable for superiors not only to hold group meetings of their own subordinates, but also to have occasional meetings over two hierarchical levels.

With short circuits, a work group now entails all possible direct contacts between a supervisor, the supervisor's immediate subordinates, and these subordinates' immediate subordinates. Likert (1961, p. 115) states that

An organization takes a serious risk when it relies on a single linking pin or single linking process to tie the organization together. ..., an organization is strengthened by having staff groups and ad hoc committees provide multiple overlapping groups through which linking functions are performed and the organization bound together.

Hence, a "tree-like" macro-structure of influential communications may arise (a) in which there are chains of sequentially overlapping zones of structural cohesion and influence and (b) in which these chains conform roughly to the shape of the authority hierarchy, as shown below:
This short circuit organization not only eliminates bridges, but also eliminates liaisons. Instead of "linking pins," the interfaces between the work groups have been thickened. We get a pattern of sequentially overlapping zones of structural cohesion and influence on the basis of which the social positions (organizational roles) can be coordinated.

8.2.3 Social Circles

Likert's concept of linking pin organization is closely related to Alba and Kudushin's concept of a social circle (Kadushin 1966, 1968; Alba and Kadushin 1976):

The social circle offers a sort a group structure appropriate for studying large-scale social organization. In contrast to the clique, with its emphasis on face-to-face interaction among all or almost all of a clique's members, the social circle's cohesion is founded on short chains of indirect interaction. Circles can be thought of as knitted out of many extensively overlapping cliques. The dispersal of overlap guarantees that influences flow readily throughout a circle's extent, as individuals who participate in more than one clique act as key points of diffusion. (Alba 1982, p. 58)

Along these lines, we might represent a macro-structure as a Venn-diagram consisting of a pattern of more or less overlapping subsets of structurally cohesive actors; see Figure 8.4. If a social circle is defined on the basis of subsets of actors who are not only structurally cohesive but also proximate in social space, then a ridge structure is implied. For Simmel (1955), a social circle is based on common affiliations on one or
more social dimensions, and therefore a social circle is most likely to encompass actors who are proximate in social space. A sequence of overlapping social circles provides a structural foundation for a ridge structure that can encompass most of the social positions in a social space.

**Figure 8.4. Intersecting Social Circles in a Two Dimensional Social Space**

As Figure 8.4 suggests, it is possible that one framework of overlapping cohesive regions of the social space may include most of the actors in a population. At the other extreme, in a segregated social space there would be multiple non-overlapping social circles, and any bridging ties between these distinct social circles would be crucial to the occurrence of flows of influence throughout the system. However, to the extent that a single interconnected framework of social circles is present, local bridges lose theoretical status as the crucial structural components of macro-level connectivity. The more systematic foundation of such connectivity is provided by the configuration of the ridge structure.

Granovetter's (1973) argument that local bridges tend to link actors who are located in different parts of a social structure is not inconsistent with the occurrence of a ridge structure that penetrates into the various occupied regions of a social space. Bridging ties are rare events and, as I show below, are anomalous phenomena from the point of view of balance theory. However, when a bridging tie occurs, it is likely to connect actors who are situated in distinct ridge structures, or actors who are at some distance from each other in the same ridge structure. Hence, as Figure 8.5 illustrates, local bridges may provide either the only basis for a flow of influence between two subsets of actors, or a basis for shorter sequences of interpersonal influence that contribute (more or less) to other influence flows.
Granovetter's (1973) arguments draw on a rich vein of theoretical development concerned with the structure of sentimental or affect relations. This work, referred to as balance theory, describes certain expected patterns of affect relations in triads and the implications of these patterns for the organization of a macro-structure of affect relations. However, it is not widely recognized that the theoretical work on balance theory has been generalized so that it is no longer restricted to affect relations, and that simplified images of macro-structure can be derived from an analysis of the pattern of interpersonal ties in the $\binom{N}{3}$ triads that are entailed in a population of $N$ actors (Johnsen 1985, 1986, 1989).

To the extent that balance theory accurately describes the structure of an observed network, a bridging tie is an anomalous or rare event, and when a bridge does occur, it is likely to join distant parts of a social structure. This assertion of balance theory is consistent with Granovetter's argument about bridges, i.e., bridges connect actors who occupy distant social positions. However, from the theoretical perspective of balance theory, there are more systematic foundations of connectivity between the parts of a social structure than bridges. These systematic foundations are dense interfaces between cliques. In balance theory these cliques are structurally equivalent positions (in a certain sense to be defined), and a ridge structure which encompasses several social positions is established by those cliques which are connected by dense interfaces.

The theoretical analysis which supports the above conclusions is presented in sections 8.3.1-8.3.7. This analysis is highly detailed, and a reader who is not interested in these details should proceed directly to section 8.3.8.
8.3.1 Elementary Concepts

Balance theory originated with the work of Heider (1946) and has been advanced in by a series of papers concerned with formal models of structures of affective or sentimental relations (Newcomb 1953, 1961, 1968; Cartwright and Harary 1956; Davis 1963, 1967, 1970; Davis and Leinhardt 1972; Holland and Leinhardt 1970, 1971, 1973, 1976; Hallinan 1974; Johnsen 1985, 1986, 1989). This generalization has been carried forward to the point that the development of macro-structural models from a triad analysis may be applied to any network $R = r_{ij}$ in which there is either a tie from actor $i$ to actor $j$ ($r_{ij} = 1$), or there is not ($r_{ij} = 0$), and $r_{ii} = 1$ for all ordered pairs of actors (compare with Freeman 1992). In the classical work on balance theory, the network of interpersonal ties took the form of a complete signed network in which the ties were equal to $+$ (indicating positive affect) or $-$ (indicating negative affect). The matrix $R$ can be used to describe the pattern of positive and negative affect relations, but it also applies to other types of social relations in which a simple interpersonal adjacency and its absence is being represented rather than the occurrence of positive and negative sentiments.

The dyads in $R$ are one of three types: mutual (M), asymmetric (A), or null (N). Actors $i$ and $j$ are M related if $r_{ij} = 1$ and $r_{ji} = 1$ ($i \leftrightarrow j$); they are N related if $r_{ij} = 0$ and $r_{ji} = 0$ (neither $i \rightarrow j$ or $i \leftarrow j$ exist); otherwise they are A related ($i \rightarrow j$ or $i \leftarrow j$).

Among the triads in $R$ there are 16 different combinations of M, A, and M dyads. Each of these combinations is a triad type that is described conventionally with three numbers $m:a:n$, where $m$, $a$, and $n$ are the numbers of M, A, and N dyads composing the triad ($m + a + n = 3$), together in some cases with a letter C, D, T, or U standing for "cyclic", "down", "transitive", and "up". The set $\Theta$ of 16 different triad types is shown in Figure 8.6. Panel (a) describes the classical representation of the triad types and panel (b) provides a simplified representation of these types.

A structural model $X$ is defined by the subset, $P_X$ of $\Theta$, of triad types that appear in the model, or by the subset $\overline{P}_X = \Theta - P_X$ of triad types that don't appear in the model. The triad types in $P_X$ are said to be permissible and triad types $\overline{P}_X$ are said to be forbidden. Not all possible subsets of permissible triads are logically consistent; for example, it is not possible to construct a network based on $P_Y = \{300,003\}$ because such a network must involve the forbidden 102 triad. Johnsen (1989) reserves the term micro-model for those subsets $P_X$ of $\Theta$ for which it is possible to construct a network in which all the possible permissible triad types appear, and he reserves the term macro-model (or macro-structural model) for the set of all the possible networks that are consistent with a particular micro-model.

In the next several sections I describe different micro-models that are constrained by transitivity and their corresponding macro-structural representations. The first two models, Balance and Clustering, appeared early in the development of this literature and describe a segregated macro-structure consisting of disconnected cliques. Generalizations of the Balance and Clustering models involve a pattern of dense interfaces between cliques. These models, described below, are the Ranked Clusters of M-cliques model, the
Transitivity model, and the Hierarchical $\tilde{M}$-cliques model. I conclude with the description of a new model, Weak Transitivity, that extends this line of generalization to a micro-model that permits all triad types which are either nontransitive (i.e., do not violate transitivity) or that satisfy transitivity in part.

**Figure 8.6. The Sixteen Triad Types**

(a) Classical representation

(b) Simplified representation
8.3.2 Balance Model

This micro-model permits only two types of triads: a triad consisting of three mutual links and triad consisting of one mutual link only:

\[ P_{BA} = \{300, 102\} \] (8.3)

The 300 triad implies the occurrence of maximal subsets of actors who are completely connected by M links; such subsets are called *M-cliques*. Now suppose we have an M-clique and an actor, Ego, who is not a member the clique. This is a possible macro-structure because all the triads in which Ego is involved are of the 102 type. Now add to this picture an actor, Alter, who also is not a member of the original M-clique. Ego and Alter must be joined by an M link (because all other possibilities are forbidden) and, in fact, any further additions to the set of actors must be members either of the original M-clique or the Ego-Alter clique, and there can be no M or A links between the cliques. In short, as shown in Figure 8.7, this micro-model implies a network that consists of at most two M-cliques that are related by N* (i.e., completely interconnected by N links).

**Figure 8.7. Macrostructure for Balance Model (Johnsen 1989)**

M completely interconnected by M
N* completely interconnected by N
8.3.3 Clustering Model

This model allows for the occurrence of three actors who have no M or A tie with each other:

\[ P_{CL} = \{300, 102, 003\} \]  \hspace{1cm} (8.4)

The resulting macro-structure, as shown in Figure 8.8, has one or more M-cliques that are related by N*.

**Figure 8.8. Macrostructure for Clustering Model (Johnsen 1989)**

8.3.4 Ranked Clusters of M-cliques Model

This model further relaxes the structural constraints and allows for A links in certain types of triads:

\[ P_{RC} = \{300, 102, 003, 120D, 120U, 030T, 021D, 021U\} \]  \hspace{1cm} (8.5)

Now all triad types are permitted in which there is a semipath connecting the three actors, so long as there is no violation of the principle of transitivity, i.e., if \( i \rightarrow k \) and \( k \rightarrow j \), then \( i \rightarrow j \) for any three actors. The following triads have a semipath connecting the three actors, but entail a violation of transitivity and, therefore, are excluded: 210, 201, 120C, 111D, 111U, 030C, and 021C. The resulting macro-structure consists of M-cliques that are arranged in levels, with one or more cliques at each level, with cliques at the same level related by N*, and with all cliques at different levels related by A* (i.e.,
completely interconnected by the A relation with all A relations going in the same direction) from the lower-level to the higher-level cliques. In this structure, there are no stand-alone cliques; see Figure 8.9.

This framework of inter-clique relations has the following noteworthy features. Two actors $i$ and $j$ who are members of the same clique are structurally equivalent in the $R = r_{ij}$ relation, i.e.,

$$r_{ii} = r_{jj}, \quad r_{ij} = r_{ji}, \quad r_{ik} = r_{jk}, \quad \text{and} \quad r_{ki} = r_{kj}$$

for all $k \neq i, j$. Hence, each clique corresponds to a distinct social position. The interfaces between cliques are either empty or complete. All the inter-clique relations are A links: if $iAj$, then actors $i$ and $j$ are in different cliques and all the members of actor $i$'s clique are A related to the members of actor $j$'s clique. There are no bridges or liaisons, so long each M-clique has two or more members. The pattern of dense interfaces is transitive; given three cliques $\{M^{(i)}, M^{(j)}, M^{(k)}\}$, if $M^{(i)} \stackrel{A^*}{\rightarrow} M^{(k)}$ and $M^{(k)} \stackrel{A^*}{\rightarrow} M^{(j)}$, then $M^{(i)} \stackrel{A^*}{\rightarrow} M^{(j)}$.

**Figure 8.9. Macrostructure for Ranked Clusters of M-cliques Model (Johnsen 1989).**

M completely interconnected by M
N* completely interconnected by N
A* completely interconnected by A
Note: All interclique \( A^* \) relations implied by transitivity are suppressed and all other missing interclique relations are \( N^* \).

### 8.3.5 Transitivity Model

This model further relaxes the structural constraints, by permitting the 012 triad:

\[
P_{TR} = \{300, 102, 003, 120D, 120U, 030T, 021D, 021U, 012\} \quad (8.7)
\]

The model is called the transitivity model because all of the forbidden triads violate transitivity and none of the permitted triads do. Triads such as 012 do not violate transitivity because a two step path \((i \rightarrow j \rightarrow k)\) does not exist. The resulting macrostructure, as shown in Figure 8.10, consists of a collection of \( M \)-cliques partially ordered by the \( A^* \) relation (by convention, every \( M \)-clique is in relation \( A^* \) to itself) where incomparable \( M \)-cliques are pairwise related by \( N^* \).

**Figure 8.10. Macrostructure for Transitivity Model (Johnsen 1989).**

Note. All interclique \( A^* \) relations implied by transitivity are suppressed and all other missing interclique relations are \( N^* \).

This macrostructure is the same as the ranked clusters of \( M \)-cliques model, except that it allows for isolated subsets of \( M \)-cliques. The structural constraints do not allow for bridges or liaisons, if every \( M \)-clique consists of two or more actors. The clique interfaces are either empty or asymmetrically complete, and the \( A^* \) relation is transitive. Each \( M \)-clique is a structurally equivalent position as defined in Eqn. (8.6).
8.3.6 Hierarchical $\tilde{M}$-Clique Model

This model further relaxes the structural constraints by allowing the 210 triad:

$$P_{HC} = \{300, 102, 003, 120D, 120U, 030T, 021D, 021U, 012, 210\} \quad (8.8)$$

The set of forbidden triads is now $\overline{P}_{HC} = \{201, 120C, 111D, 111U, 030C, 021C\}$. The result is a macro-structure with the same restrictions on the framework of inter-clique relations that was found in the transitivity model, but in which the M-cliques are replaced by $\tilde{M}$-cliques. An $\tilde{M}$-clique is a strong component in the subnet of $R$ that is obtained by eliminating the A ties; i.e., all the pairs of actors in an $\tilde{M}$-clique are connected by a path of M ties of some length. The resulting macro-structure, is the same as that shown in Figure 8.10, except that the M-cliques are replaced by $\tilde{M}$-cliques.

In this macro-structure, every dyad in an $\tilde{M}$-clique is either M or A related. The subnet of M and A relations within a clique does not need to be transitive (although the subnet of A relations within such cliques is). The members of an $\tilde{M}$-clique are structurally equivalent with respect to their profiles of ties sent to and received from actors in other cliques; but they are not necessarily structurally equivalent in terms of their profiles of ties with actors in the same clique. Thus, two actors $i$ and $j$ in an $\tilde{M}$-clique are structurally equivalent in $R = r_{ij}$ in the weaker sense of having identical profiles of interpersonal contacts ignoring whether these contacts are based on an M or A relation:

$$r_{ii}^* = r_{ij}^*, \quad r_{ij}^* = r_{ji}^*, \quad r_{ik}^* = r_{jk}^*, \quad \text{and} \quad r_{ki}^* = r_{kj}^*$$

for all $k \neq i, j$, where $r_{ij}^* = \max(r_{ij}, r_{ji})$.

The hierarchical $\tilde{M}$-cliques model is the most general of the models thus far considered. However, in an analysis of the networks in the Davis-Leinhardt data set (Davis 1970; Davis and Leinhardt 1972), Johnsen (1989) shows that this model does not adequately deal with large groups (sizes 39-79 in the Davis-Leinhardt data), and he develops a model that appears to handle these large groups. Unlike the hierarchical $\tilde{M}$-cliques model, in Johnsen's large group model the 120C triad is permitted and the 003 is prohibited:

$$P_{39} = \{300, 102, 120D, 120U, 030T, 021D, 021U, 012, 210, 120C\}.$$  

Permitting the 120C triad entails a further relaxation of structural constraints along the same lines of the successive relaxation of constraints that we have been pursuing. However, Johnsen's large group model also reintroduces the prohibition of the 003 triad, that had been permitted earlier (with the clustering model) in the sequence of generalizations. Reintroducing the 003 triad entails an unsatisfying break in the theoretical line of generalization; moreover, especially in large groups the possibility of three N linked actors should not be forbidden. Therefore, I present a modified version of Johnsen's large group model in which the 003 is not forbidden.

8.3.7 Weak Transitivity Model

This model relaxes the constraint of the hierarchical $\tilde{M}$-cliques model by allowing the 120C triad:
The set of forbidden triads is now \( \Phi_{\text{wt}} = \{201, 111D, 111U, 030C, 021C\} \). For convenience, I display these forbidden triads below:

![Forbidden Triads Diagram]

The common feature of these forbidden triads is that each possesses the conditions of transitivity (one or more two-step paths) and does not satisfy transitivity at all; i.e., the intransitivity is maximal. In the previous models, a triad was forbidden if it contained any evidence of intransitivity; hence, the 120C triad (where \( i \rightarrow k \rightarrow j \) and \( i \leftrightarrow j \)) is intransitive. The weak transitivity model permits all triads that either are non-transitive (do not entail the conditions of transitivity and therefore cannot be intransitive) or that contain any evidence of a satisfaction of the conditions of transitivity. Hence, this is the most general of the models of macro-structure in which transitivity constrains, to some extent, the pattern of social ties.

In this weak transitivity group model, the macro-structure consists of \( \tilde{M} \)-cliques, and each \( \tilde{M} \)-clique has only M and A ties. Because 120C is now permitted, transitivity may be violated within \( \tilde{M} \)-cliques. The interfaces between the cliques are either empty or complete. Unlike the previous models, a clique interface is not necessarily anti-symmetric; i.e., there may be A links in both directions on a single interface. The clique interfaces in this model are either N*\( \tilde{M} \), A*, or A#, where A# is an interface in which, for all actors \( i \) and \( j \) who are in different cliques, either iAj or jAi. Thus, A* is a special case of A#; for two cliques, \( \tilde{M}^{(i)} \) and \( \tilde{M}^{(j)} \), if \( \tilde{M}^{(i)} \xrightarrow{A*} \tilde{M}^{(j)} \), then \( \tilde{M}^{(i)} \xleftarrow{A*} \tilde{M}^{(j)} \).
In this weak transitivity model, for every three distinct $\tilde{M}$-cliques $\{\tilde{M}^{(i)}, \tilde{M}^{(j)}, \tilde{M}^{(k)}\}$, if $\tilde{M}^{(i)} \xrightarrow{A^*} \tilde{M}^{(k)}$ and $\tilde{M}^{(k)} \xrightarrow{A^*} \tilde{M}^{(j)}$, then $\tilde{M}^{(i)} \xrightarrow{A^*} \tilde{M}^{(j)}$; that is, $A^*$ is transitive on the $\tilde{M}$-cliques. However, unlike the hierarchical $\tilde{M}$-cliques model in which the interfaces between cliques were either $N^*$ or $A^*$, in this model an interface may be $A^#$ or $N^*$, where it is not forced by transitivity to be $A^*$.

In short, as Figure 8.11 shows, we have a macro-structure consisting of subsets of $\tilde{M}$-cliques. Each of these $\tilde{M}$-cliques corresponds to a structurally equivalent position in the weak sense described by Eqn. (8.9). Besides the $\tilde{M}$-cliques, there also may be framework of dense interfaces between cliques; these interfaces may or may not join all the cliques. There are no bridges or liaisons, so long as each $\tilde{M}$-clique has two or more members. What is remarkable about this development is that we have now reduced the set of prohibited triad types to five types, on which basis we must conclude that the systematic basis of inter-clique connections must be the $A^#$ relation, and that bridges and liaisons can only exist if they involve an actor who is a clique unto him or her self, i.e., who has no mutual ties with any other actor.

**Figure 8.11. Macrostructure for Weak Transitivity Model.**

Note. All interclique $A^*$ relations implied by transitivity are suppressed and all other missing interclique relations are $N^*$.

### 8.3.8 Ridge Structure in Transitive Macro-Models

I have shown that the idea of a ridge structure (sequentially overlapping, densely occupied, regions of social space in each of which the probability of attachments among
actors is high) has appeared in various forms in the literature. Among these realizations of the concept of ridge structure, the macro-structural models related to balance theory are the most suggestive. We have seen that a ridge structure is consistent with the tendency toward transitive closure. The models of balance theory do not require ridge structures that encompass all the actors of a population; for example, segregated structures composed of disconnected cliques are permitted. However, balance theory suggests that interactions between cliques are not idiosyncratic in that the interfaces between cliques are either filled with ties or they are empty.

Hence, bridges and liaisons are structural anomalies or rare events to the extent that maximally intransitive triadic structures do not occur in a population. To be sure, we may find a bridge or liaison involving actors who are members of one-actor cliques; however, if there is a systematic basis of structural connectivity among cliques, then it is likely to take the form of a framework of dense interfaces between cliques.

Moreover, in the macro models that are consistent with transitive closure, a clique is also a social position. If two actors are said to be in contact when they are joined by a semipath of length one, then all pairs of members of a clique will be in such contact with each other and they will have an identical profile of contacts with members of other cliques. Hence, cliques will consist of structurally equivalent actors in the weak sense described by Eqn. (8.9). In turn, it follows that these macro-structural models can alternatively be represented as blockmodels, with the understanding that the structural equivalence of the members of a social position (clique) is based on the semipath structure of the network.

An additional implication of this perspective is that vacuous social positions (in which there are no interpersonal ties) are a theoretical anomaly. To be sure, the measure of structural equivalence permits as a logical possibility the occurrence of a subset of actors who are structurally equivalent and who do not have any direct ties among themselves; however, the theoretical expectation is that social positions will be filled with ties. Dense social positions are implied by the prohibition of maximally intransitive triad structures. Moreover, if shared social positions (clique memberships) reflect shared positions in social space, then dense social positions are to be expected.

8.4 Concluding Remarks

Ridge structures arise because of strong associations among the dimensions that define the social space of a population of actors and that affect the occurrence of interpersonal attachments. Hence, ridge structures entail a non-random distribution of interpersonal attachments, and these attachments, in turn, govern the distribution of social influence and the integration (via interpersonal agreements) of a complexly differentiated population. In this closing section of the chapter, I discuss the relationship of this argument about ridge structures with Blau’s work on the structure of social space (Blau 1977, 1994; Blau and Schwartz 1984).

Blau’s (1977, 1994) analysis of social space revolves on two assumptions,

(a) the probability of social ties depends on opportunities for contact, and

(b) proximity in social space increases the probability of social ties,
and an attribute—consolidation—of the set of dimensions that define a social space. Consolidated dimensions are perfectly associated, whereas unconsolidated (intersecting) dimensions are orthogonal. Blau's argument is that unconsolidated dimensions foster social ties between actors who differ on some of the dimensions that define the social space, and that such ties are the foundation of the integration of a complexly differentiated population; much the same argument has been made by Laumann and Knoke (1986, p.86). I will not draw on Blau's first assumption; it is not necessary for the developments that I will pursue, because the structural conditions that affect opportunities for contact are taken into account in the definition of social space.\(^{41}\) It is the second assumption that is crucial.

For his argument, Blau relies on the principle of homophily. For example, under homophily, persons of similar age and wealth are more likely to interact than persons of dissimilar age and wealth. In a consolidated structure, persons who are similar in age also are likely to be similar in wealth. Consequently, the propensity toward age-peer interaction reinforces the propensity toward wealth-peer interaction. In this situation, interpersonal ties should occur disproportionately among homogeneous categories of actors (e.g., among young poor persons and among old wealthy persons) and between the most proximate age-wealth categories. In an unconsolidated situation, because wealth and age are not associated, interaction with wealth peers will weaken the tendency to interact with age peers. Similarly, interaction with age peers will weaken the tendency to interact with wealth-peers.

In an unconsolidated situation, actors will be randomly dispersed in social space, and so will their interpersonal ties (Blau 1994: 125). Blau suggests that a decrease in the consolidation of two determinants of interpersonal relations diminishes their salience. For example, the weaker the association between age and wealth, the less salient either factor will be in determining the occurrence of interpersonal relations. It follows that in an unconsolidated situation, homophily is attenuated and interpersonal ties are randomly distributed among social positions that are themselves randomly distributed in social space (because the dimensions are orthogonal).

If dimensions are perfectly consolidated, then an actor's location on any one dimension predicts his or her location on each of the other dimensions, and two actors who have an identical status on one dimension will have an identical status across all dimensions. Such consolidation is consistent with the formation of a ridge structure that entails sequentially overlapping regions of structural cohesion in social space. In consolidated social spaces, where the probability of social ties declines precipitously with social distance, interpersonal ties will be non-randomly distributed in the social space and will tend to be confined to relatively tight social manifolds.

For instance, consider a social space that is defined only by income and education, and in which these two dimensions have a strong linear association. In this simple 2-dimensional social space, the actors are distributed approximately in a straight line: each actor occupies a location that might be shared with other actors, and he or she has neighboring positions on one or both sides that may be far or near, and that may also be occupied by one or more actors. We might observe a segregated clustering of the actors in which there are few social ties between the clusters; i.e., all the social ties will be among
actors of similar status on education and income. However, it is important to see that this formulation also allows for a situation in which all actors (apart from the those in the most extreme social positions) are sequentially arrayed such that each has proximate nearest neighbors on either side. Because of the strong association between education and income, such a distribution of actors implies the presence of a tight social manifold that is comprised of a sequence of social positions in which the probability of social ties between contiguous positions is high. This is the elemental social structural foundations of a ridge structure. If a sufficient number of actors are distributed in a sequential pattern (so that there is a continuum of overlapping and densely occupied regions of social space), then the tendency to form ties with actors who are proximate in social space will produce a framework of structural cohesion that includes the entire population.  

Blau recognizes the possibility of ridge structures; for example, he writes (1974: 624), "Whereas status diversity does not make interpersonal relations between the highest and lowest strata likely, it links them indirectly by fostering personal relations between strata that are not far apart." However, he argues (1974: 625) that such connectivity is less important than the inter-group ties that are fostered by homophily on single social dimensions. "Status diversity contributes to macro social integration, as heterogeneity does, though not as much, because it furnishes only indirect links between social strata that are widely separated."

I argue that the indirect connectivity that is entailed in a ramifying ridge structures is a crucial structural foundation of macro-integration. Blau's analysis of heterogeneity gets into theoretical trouble because he ignores the social distance between actors in the multivariate social space when describing the mechanism by which social ties are formed between actors who differ on one or more of the dimensions. If actors are located in social space, then they are separated by a certain distance in that space and it is that distance which governs the probability of a social tie between them. If two actors, who are distant from each other in social space, happen to share an affiliation with some group, then such affiliation (when it has been taken into account in the locations of the actors) should not alter the probability of a social tie; this probability will be low regardless of any shared affiliations if the actors are sufficiently distant in social space. If two actors have proximate, nonidentical, positions in social space, then they are likely to be similar on many but not all of the dimensions that define the space; their proximity in social space entails a high probability of a social tie, and this social tie, because it occurs between actors in nonidentical positions may involve a crosscutting of group boundaries (i.e., the two actors may be in different conditions of a nominal classification). In short, a preference to associate with persons of like characteristics is constrained by social distance.

Blau's analysis does not address the overall configuration of social ties in the social space. The occurrence of crosscutting social ties, constrained by social distance, implies a foundation of macro-level social integration, but does so only in a limited way. Ties between actors in proximate (nonidentical) social positions is a necessary constituent of social process that form agreements between actors in distant locations in the social space. However, these ties may not produce a connectivity that includes the entire population of the social space. If social ties are not likely between actors who are distant
from each other in social space, then macro-level integration must rest on the pattern of indirect connectivity among social positions.

Finally, Blau does not address the causal connection between the macro-structure of social ties and outcomes that integrate (in some concrete sense) the actors and positions. He suggests, as do Laumann and Knoke (1986), that a population will be integrated to the extent that its dimensions are unconsolidated. Unconsolidated dimensions imply a random distribution of actors in social space. Because the actors are randomly distributed in the social space, both social stratification and ridge structures are unlikely events. Hence, Blau’s argument associates integration with the absence of social differentiation, and he does not address the social processes by which concrete integrative outcomes (such interpersonal agreements and coordinated action) may be achieved within the context of a highly differentiated social structure.

I forward an hypothesis that is different from Blau’s. Highly consolidated social dimensions, in a population of sufficient size, provide densely settled regions of social space, and (depending on the distribution of actors in the space) they also provide the basis of a ridge structure by which actors in distant locations are joined by sequential interlocking zones of structural cohesion. Such a ramifying ridge structure (which penetrates into distant regions of social space and includes most of the positions in the space) is a basis for the production of interpersonal agreements between actors who occupy distant social positions; ridge structures not only establish a robust basis of connectivity, but also (because most self-weights are diminished by a high mean indegree), enable the production of agreements via a social influence process. Hence, social differentiation segregates persons by putting social distance between actors; but, while segregating actors, social differentiation also may provide a structural basis for boundary-spanning interpersonal agreements.

I do not assert that ridge structures, which allow the formation of consensus in a population, are ubiquitous or even that they are frequent. What I assert is that consensus is based on ridge structures and that the characteristics of a ridge structure determine the pattern of interpersonal agreements that get formed. The characteristics of a ridge structure depend on the distribution of actors in social space: the greater the proximity of the actors in a particular region, the greater the density of ties; and the greater the inclusiveness of the social manifold (its penetration into all the occupied regions of the social space), the greater the likelihood of a ridge structure that encompasses the entire population. Thus, a ridge structure can take the form of segregated subgroups between which few, if any, social ties exist and it can take the form of a coherent framework of social cohesion that penetrates into all the occupied regions of the social space and that encompasses the entire population. In the former case, agreements must be predicted on social choice mechanisms, because the structural foundations of social influence processes are absent. In the later case, the ridge structure allows a production of interpersonal agreements between the distant social positions that it includes.

I develop this argument in the next two chapters. First, in Chapter 9, I show that social influence processes, when played out in a ridge structure, can produce agreements among actors in complexly differentiated social structures. Second, in Chapter 10, I show that in large-scale complexly differentiated populations the production of consensus, via
social influences, is associated with the occurrence of a dominant position-cluster, and I describe the structural foundations of such dominance.

8.5 Endnotes

34Granovetter (1973) develops the concept of local bridge. Unlike a bridge, when a local bridge is removed from a network the members of the local bridge (and their contacts) remain joined in the network. The "degree" of a local bridge is the length of the shortest path which joins the members of a local bridge, were the local bridge removed from the network. Granovetter points out that as the degree of a local bridge increases, bridges and local bridges may tend to become equivalent in terms of their roles in networks; for example, influence will not tend to flow over very long paths, so that a local bridge of high degree or a bridge may be the only effective channel of such flow between two persons and their direct contacts. Local bridges have a minimum degree, which is three; thus, directly joined dyads with one or more common contacts (paths of length two) are not local bridges.

35If the strength or weakness of a tie is based on criterion other than the probability of its occurrence, then strong bridges are not entirely precluded (Friedkin 1982, p. 285). The importance of local bridges is the connectivity they provide between different parts of the macro-structure. This is most clear in a formal hierarchy of authority composed of strong interpersonal ties; in such a macro-structure every interpersonal tie is a strong bridge.

36Ridge structures include as special cases the occurrence of disconnected internally cohesive subgroups and segmented structures in which such subgroups are connected by bridging ties. In these cases, the ridges are the subgroups themselves.

37Similarly, with respect to a valued social relation, the variance of the values on the lines should be zero for each subset of lines within and between positions containing structurally equivalent actors.

38Whether the macro-structure of the population can be condensed into a simple image of the ridge structure, depends on the number of distinct social positions in the population. To the extent that a large population is partitioned into a small number of distinct social positions, a simplified image of the macro-structure is possible. Moreover, given information on the number of occupants of each social position, the blockmodel image is a complete representation of the network of social ties.

39Hence, peripheral actors in a center-periphery pattern, because they are one-actor cliques, occupy different social positions.

40In my approach to influence networks and social spaces, if the actors within \( \tilde{M} \)-cliques are structurally equivalent as in Eqn. (8.6), then they also will be structurally equivalent in the influence network and they will have identical positions in social space. The macro-structure of cohesion is a direct description of the pattern of interpersonal ties; the influence network and social positions are derived from this structure. If the actors within \( \tilde{M} \)-cliques are structurally equivalent as in Eqn. (8.9), then the actors who belong to a common clique may not occupy an identical location in social space; however, it likely that they will occupy proximate positions.

41If an opportunity for contact is simply a nonzero probability of a social tie and, if the association of social distance and social ties is not contingent on other variables, then the analysis of the distribution of social ties can focus strictly on the social distance between actors.

42The situation is complex, but not fundamentally different in a social space that is defined by many dimensions that vary in the associations among them. Depending on the character of these associations and the size of population, actors will be distributed in social space. Whether the social space is organized in a fashion that permits the emergence of a ridge structure depends on the existence of relatively tight social manifolds, each containing a contiguous zone of densely occupied regions of social space. It is the multivariate distribution of actors that determines the existence and configuration of such manifolds.

43In an unconsolidated situation, where the forces determining the occurrence of interpersonal relations are attenuated, interpersonal relations should be equally likely among all the pairs of members of the group. This implies that for any partition of the group into subgroups (obtained through a simultaneous rearrangement of the rows and columns of the adjacency matrix), the network densities within a between
particular subgroups are expected to be uniform. In effect, in an unconsolidated structure there are no subgroups, if we define the occurrence of a subgroup in terms of a differential in the density of relations within and between subsets of group members. The pattern should be random.

44 A random distribution does not necessarily imply low level of structural cohesion, given a sufficiently high density of social ties. However, if weak associations also imply weak pressures to initiate a social tie, then the average number of social ties will decrease as consolidation decreases. Moreover, as the dimensions of social space become less salient, stratification of social ties will become less pronounced.